WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2020, held in 2021

MTMGDSE01T-MATHEMATICS (DSE1)

MATRICES

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Write short note on Linear independence of vectors.

(b) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 5 & 2 \\ 4 & 8 & 0 \end{bmatrix}$.

(c) Show that for two non-singular matrices A and B of same order $(AB)^{-1} = B^{-1}.\, A^{-1}$.

(d) State Cayley-Hamilton's theorem.

(e) Show that the matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is an orthogonal matrix.

(f) Show that $S = \{(x, y, z) \in \mathbb{R}^3 : 3x - 4y + z = 0\}$ is a sub-space in $\mathbb{R}^3$.

(g) Show that the vectors $\alpha_1 = (0, 2, -4)$, $\alpha_2 = (1, -2, -1)$, $\alpha_3 = (1, -4, 3)$ are linearly dependent.

(h) Write a simple 3×3 matrix whose all eigen values are 1, 2, 3 respectively.

(i) When a matrix is not invertible?

(j) Write the equations in matrix from $x_1 = x\cos\alpha + y\sin\alpha$ and $y_1 = -x\sin\alpha + y\cos\alpha$.

2. (a) Examine whether the set S is a subspace of $\mathbf{R}_3$ or not, where 4

$$S = \{(x,\ y,\ z) \in \mathbf{R}_3 \mid x = 0\}$$

(b) If $\alpha = (1,\ 1,\ 2)$, $\beta = (0,\ 2,\ 1)$, and $\gamma = (2,\ 2,\ 4)$, determine whether they are linearly independent or not. 4

3. (a) If $A=\begin{bmatrix}1 & 3 & 0\\ 3 & 7 & 2\end{bmatrix}$, $B=\begin{bmatrix}2 & 5 & 8\\ 1 & 3 & 2\end{bmatrix}$, $C=\begin{bmatrix}1 & 3\\ 2 & 4\\ 1 & 5\end{bmatrix}$, then establish that $(A+B)C=AC+BC$. 4

(b) If $P=\begin{bmatrix}6 & 12 & 13\\ 14 & 24 & 25\\ 10 & 16 & 18\end{bmatrix}$, and $Q=\begin{bmatrix}11 & 8 & 3\\ 13 & 9 & 15\\ 14 & 21 & 18\end{bmatrix}$ then establish 4

(i) $(P+Q)^T=P^T+Q^T$ and (ii) $(P.Q)^T=Q^T.P^T$

4. (a) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$, where 3+1

$$W=\{(x,\ y,\ z)\in R^3\ :\ x+y+z=0\}$$

(b) If $A+I=\begin{bmatrix}1 & 3 & 4\\ -1 & 1 & 3\\ -2 & -3 & 1\end{bmatrix}$, evaluate $(A+I)(A-I)$, where I represents the 3×3 identity matrix. 4

5. (a) Find the inverse of the matrix $\begin{bmatrix}2 & 1 & 1\\ 1 & -1 & 0\\ 2 & 1 & -1\end{bmatrix}$ and use it to solve the following system of equations: 2+2

$$2x+y+z=5$$
$$2x+y-z=1$$
$$x-y=0$$

(b) Solve by matrix method: 4

$$2x-y=1$$
$$x+y=2$$

6. (a) Find the eigen values of the matrix $\begin{bmatrix}1 & -1 & 2\\ 1 & 1 & 1\\ 1 & 3 & 1\end{bmatrix}$. 4

(b) Prove that if λ be an eigen value of a non-singular matrix A, then λ^{-1} is an eigen value of A^{-1}. 4

7. (a) Prove that two eigen vectors of a square matrix A over a field F corresponding to two distinct eigen values of A are linearly independent. 4

(b) Prove that the eigen values of a real symmetric matrix are all real. 4

8. (a) Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 3 & 4 & 5\\ 4 & 6 & 8\end{bmatrix}$. 4

   (b) Define elementary matrix. Also show that elementary matrices are non singular. 4

9. (a) Prove that a matrix is non-singular if and only if it can be expressed as the product of a finite number of elementary matrices. 4

   (b) Prove that if the rank of a real symmetric matrix be 1 then the diagonal elements of the matrix cannot be all zero. 4

10.(a) Use Cayley-Hamilton theorem to find A^{100}, where $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$. 4

   (b) Show that $B=\{(1,\ 2,\ 1),\ (0,\ 1,\ 0),\ (0,\ 0,\ 1)\}$ is a basis of $\mathbb{R}^3$. Express the vector $(1,\ 2,\ 3)\in\mathbb{R}^3$ as a linear combination of the basis B. 1+3

11.(a) Reduce the matrix to the fully reduced normal form 4

$$\begin{bmatrix}1 & 0 & 2 & 3\\ 2 & 0 & 4 & 6\\ 3 & 0 & 7 & 2\end{bmatrix}$$

   (b) Find all real matrices $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, such that $A^2=I_2$. 4

12.(a) If $A=\begin{bmatrix}a & b & c & d\\ -b & a & d & -c\\ -c & -d & a & b\\ -d & c & -b & a\end{bmatrix}$, compute AA^t. 4

   (b) Find matrix A, if $\operatorname{adj} A=\begin{bmatrix}1 & 3 & -4\\ -2 & 2 & -2\\ 1 & -3 & 4\end{bmatrix}$. 4

13.(a) Find the equation of the line through the following pair of points in (3, 7, 2) and (3, 7, –8). 2

   (b) Find the equation of the plane containing the following point in space: 2

$$(1,\ 1,\ 1),\ (5,\ 5,\ 5)\ \text{ and }\ (-6,\ 4,\ 2)$$

   (c) Prove that the set $S=\{(1,\ 0,\ 1),\ (0,\ 1,\ 1),\ (1,\ 1,\ 0)\}$ is a basis of $\mathbb{R}^3$. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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