WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2019

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Find an integrating factor of the differential equation $y^2 + (x - \frac{1}{xy})\frac{dy}{dx} = 0$. 2

(b) What is the Clairaut's form for first order ordinary differential equation? Write down the general solution of it. 2

(c) Find the Wronskian of x and $-x$ in $(-1, 1)$. 2

(d) Find $\frac{1}{(D-1)^2}(x^2e^{3x})$, where $D = \frac{d}{dx}$. 2

(e) Eliminate the arbitrary function f and F from the relation $y = f(x-at) + F(x+at)$. 2

(f) Determine the order, degree and linearity of the following PDE: 2

$$xy\left(\frac{\partial^2 z}{\partial x^2}\right)^2 - 2\frac{\partial z}{\partial y} = 1$$

(g) Classify the following partial differential equation: 2

$$z_{xx} - 2\sin x\, z_{xy} - \cos^2 x\, z_{yy} - \cos x\, z_y = 0$$

(h) Verify the condition of integrability for the equation 2

$$(2x + y^2 + 2xz)\,dx + 2xy\,dy + x^2\,dz = 0$$

2. (a) Examine whether the following differential equation is exact and if so find the general solution. 4

$$(\cos y + y\cos x)\,dx + (\sin x - x\sin y)\,dy = 0$$

(b) Obtain the general solution of the differential equation 4

$$\frac{d^2y}{dx^2} + 4y = \sin^2 x$$

3. (a) Solve the following differential equation 5

$$(px-y)(x-py)=2p \text{ where } p=\frac{dy}{dx}.$$

(b) Prove that x, x^2 and x^4 are independent solution of the differential equation 3

$$x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=0$$

Write down the general solution also.

4. (a) Solve: $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^2e^{3x}$ 4

(b) Solve: $(x^2D^2-xD+4)y=\cos(\log x)+x\sin(\log x)$, where $D=\frac{d}{dx}$. 4

5. (a) Solve: $\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0$; $\frac{dy}{dt}+5x+3y=0$ 4

(b) Solve $(D^2+2D+1)y=e^{-x}\log x$ by the method of variation of parameters. 4

6. (a) Solve: $\frac{a^4dx}{(b-c)yz}=\frac{b^3dy}{(c-a)zx}=\frac{c^2dz}{(a-b)xy}$ 5

(b) Find particular integral of the differential equation $(D^2+49)y=x\sin x$, where $D\equiv\frac{d}{dx}$. 3

7. (a) Eliminate a, b from the relation: 3

$$z=ax^2+by^2+ab$$

(b) Solve the partial differential equation by Lagrange's method: 5

$$y^2(x-y)p+x^2(y-x)q=z(x^2+y^2)$$

8. (a) Find a complete integral of the following partial differential equation by Charpit's method: 5

$$pxy+pq+qy=yz$$

(b) Form a partial differential equation by eliminating the arbitrary function from the relation: $x+y+z=f(x^2+y^2+z^2)$ 3

9. (a) Solve: $(x^2+y^2+z^2)dx-2xy\,dy-2xz\,dz=0$ 4

(b) Solve: $(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x(1-x^2)$, given that $y=x$ is a solution of its reduced equation. 4

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